Optimal Weapon and Armor for First-level Fighter in D&D 3e

Ethan Kennerly while collaborating with Steve Borst. August 2004.

An Algebraic Algorithm

In simple situations, or a small problem size, a brute force search is sufficient to generate a fast result. If there are an average of n items per location, and k location, then brute force search of items is $\Theta(n^k)$ run time complexity and it is parallelizable, except the comparison of evaluations.

If an algebraic formula of item property values describes the optimum, then the problem is a variation of the known knapsack problem: which selects maximum density. Maximal density can be calculated in $\Theta(kn)$ running time, and $\Theta(k)$ space complexity. But if the optimal is not a monotonic combination, such as if there are combinations that are independently optimal but collectively suboptimal, then this knapsack solution will return an inferior result.

In the case of equipment selection for an average human with no class for a survival series of melee combat trials against an unarmed opponent, optimality is a function of maximum damage infliction rate and minimal damage reception. A simple formula with infinite money and encumbrance:

> hands: select maximum damage rate.
> all others: select maximum defense rate.

Maximum damage rate, for a probability distribution is the expected value, or average. However it might be wisest to select with the highest 25th percentile in a pessimistic case. 2d6, thus would be preferable to 1d12+1, although the latter has a higher average ($7 < 7.5$), the former has a higher 25th percentile ($5 > 4$). Comparing worst cases, instead of average cases, would be undesirable, since: 1d2+2 is clearly inferior to 1d10. The 25th percentile is an arbitrary caution index. Maybe a gambler prefers the 50th percentile, which in symmetric distributions is identical to the average.

A simple linear approximation, using only integer constants is:

Approximate Expected Damage

Let W be the attacker's weapon.
Let N be the number of damage dice of W.
Let S be the sides on each damage die of W.
Let T be the lower bound of the critical threat range of W.
Let M be the critical threat multiplier of W.
Let A be the number of sides on the attack die, which is 20.
Let X be the attacker's bonus to hit.
Let Y be the defender's armor class.
Let P be the probability of X hitting Y:

$$P = (1 + A + X - Y) / A$$

Let B be the basic average damage of W:

$$B = N (1 + S) / 2$$

Let R be the critical threat probability of W:

$$R = (1 + A - T) / A$$

Let C be the critical damage coefficient of W:

$$C = 1 + M R$$

Let D be the average damage per hit of W:

$$D = B C$$

Let E be the expected damage of W by X against Y:

$$E = D P$$

Although there are several variations of critical threat ranges and critical threat multipliers and dice combinations, many of these differences average out to have identical expected damages. For example: Bastard Sword, Dwarven War Axe, and Halberd have identical average damage per hit. So do Heavy Pick, Rapier, and Scimitar. Within 1%, so do: Falcion, Great Club, and Scythe. This is only approximate for small critical threat probability.

Each item can be analyzed algebraically, according to a linear equation that approximates the expected damage rate for the item against every expected armor class. For example of the Bastard Sword, a chart could be created.

The solid line represents the statistical data of expected damage per attack. The circles locate the linear approximation, which is accurate for all D&D weapons configurations, with armor classes between 2 and 20, to a margin of error less than 2%. Accuracy of the approximation requires restricting the simulation to direct duel against a single opponent with no skills, initiative, or modifications of any kind other than the basic melee attack.

Melee Duel Damage Efficiency

This can be combined with armor to form melee efficiency. Review above, the expected damage, e, is the product of probability of hit, p, and the expected damage per hit, d.

x: attack bonus of offender
y: armor class of defender
$p = (1 + a + x - y) / a$

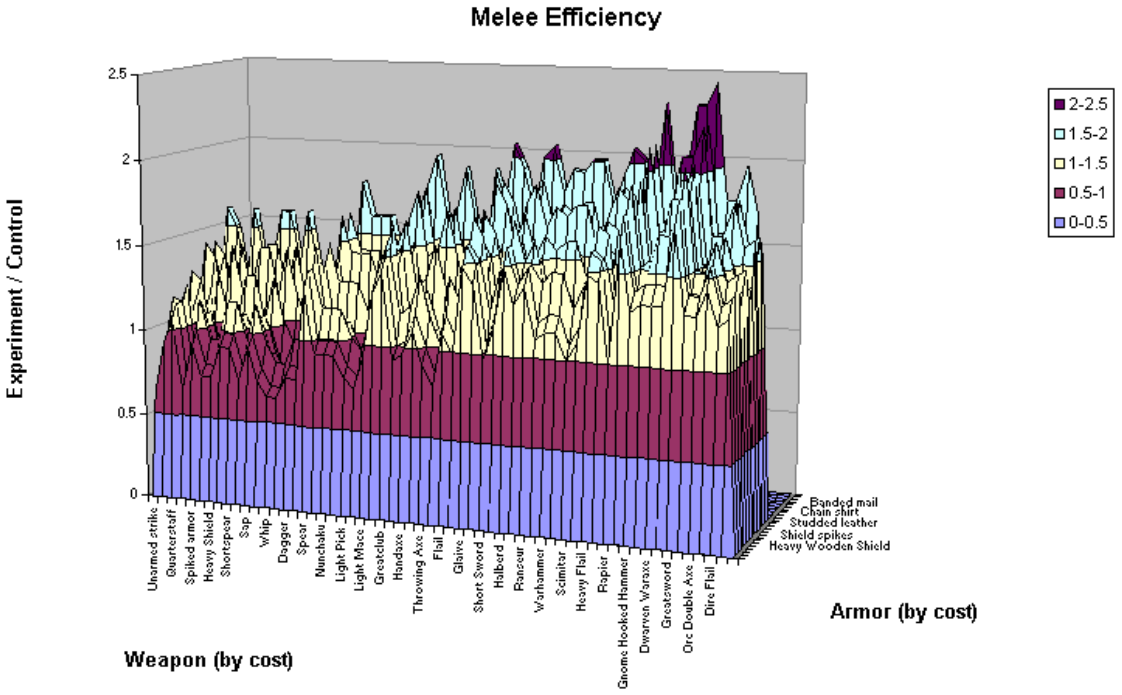
So the expected damage also varies depending on the defender's armor class.

$e = p\ d$

The melee efficiency, for a given pair of combatants can be computed. Equip each combatant in the duel, combatant number 0 and combatant number 1. Having done so, their vital attributes may be calculated that provide the formula for their expected damage. If the combatants are identical except for equipment and prerequisite equipment feats, then the rest of their attributes are identical, except equipment dependent attributes: damage and armor class. The armor class of 0 affects the probability to hit of 1 and vice versa.

$f = e0 / e1$

A chart of all possible weapon and armor pairs, with a shield being assumed if valid, produces a surface of melee efficiency. If the articles are sorted by cost this appears as a mountainous slope that gradually increases, yet has many saddles and draws. With a budget constraint, the back end of the mountain forms a sheer cliff face, as the articles outside of the budget are arbitrarily set to yield 0 melee efficiency.



Comparing damage rates for a given set of equipment and armor provides optimum equipment. If there were no budget restriction, and only one piece of armor allowed, then the maximum combination is Full Plate and a Greatsword. For a budget constrained to 150 gold pieces, then the maximum combination is **Scalemail and Greatsword**. This has a slightly higher damage delivery to damage reception ratio than Scalemail and Greataxe, or Scalemail, Heavy Wooden Shield, and Bastard Sword or Dwarven War Axe. The Player's Handbook (3.5e) provides a human starting package. For the starting gear, *Scalemail and Greatsword* are the primary items.

This analysis does not take into account differences for initiative, feats, proficiencies, reach, tactics, or terrain of the melee encounters. It as if two opponents standing five feet away from each other were told to attack each other without moving from their spots or deviating from their melee. Therefore, some specialized weapons, such as the polearms show lower efficiency, although they have specialized uses. Weapons with alternate purposes, such as Sap for subdual, also have unfavorable results.

Some Rates of Change

How much is an extra point of attack bonus worth? Attack bonus affects the expected damage, as the preceding equations displayed:

Let P be the probability of X hitting Y:

$$P = (1 + A + X - Y) / A$$

Let B be the basic average damage of W:

$$B = N (1 + S) / 2$$

Let R be the critical threat probability of W:

$$R = (1 + A - T) / A$$

Let C be the critical damage coefficient of W:

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The rate of melee efficiency per marginal attack bonus, within a range of influence, equals the sides on an attack die and the total attack bonus minus the armor class of the target plus one.

The derivative per marginal armor class may also be taken. In this case, one is concerned about one's own armor class, not the opponent, so the opponent's probability to hit must be modified.

Melee Duel Win Ratio

Melee efficiency is not sufficient to account for asymmetric factors outside of trading blow for blow. For example, a player with higher hit points may have equal melee efficiency, but will tend to win the fight. To model this, a win ratio exists, which is based on the factors of melee efficiency and health ratio.

f: melee efficiency;
l: health ratio;
w: melee win ratio;

$l0 = h0 / h1;$
$w = f0\ l0;$

Another factor is initiative. This at least determines how a tie may be broken since the first combatant to fall cannot strike back. So the value that initiative has in melee combat can be summarized as the value of not being hit. The chances of this damage not being applied times the damage provides a bonus. Whereas the chances of being hit and falling oneself from the damage tends to cancel this out.

The number of rounds depends on the damage being done. So this can be represented as the expected damage for the last round times the probability of striking first on the last round. Since both players roll for initiative, the probability has a non-uniform distribution, analogous to rolling two six-sided dice. It is two twenty-sided dice. A summation represents this probability.

This affects the lifespans of the combatants. So the ratio of melee duel lifespans, accounting for initiative, becomes:

For these values, the plot of armor class (+11), attack bonus, and health (+10) reveals that armor class has the greatest asymptotic growth, then damage bonus, then attack bonus, then health, and finally initiative.

All attribute plots intersect at (0,1) which means that no advantage there is a melee duel win ratio (w0) of 1. Armor class has the most striking asymptotic growth. Its extreme divergence from other factors suggests an error. The higher one's armor class, the lower an opponent's expected damage becomes. At an advantage of 5 armor class, the opponent's chance to hit drops from 50% to 25%. All other factors remaining equal, this suggests a doubling of the melee duel win ratio.

The power attack feat has a global optimum, since it subtracts attack and adds damage. The expertise feat apparently has no net effect in this scenario, since it subtracts attack and adds armor class.

Initiative has particularly of low asymptotic growth, because the expected damage being done each round is about 1/5 of the total damage capacity. Therefore the effect of striking first in the last round at most changes by 1/5 of total damage capacity. Probability to hit has a ceiling in OGL rules of 95%, so the attack bonus becomes useless above this level.

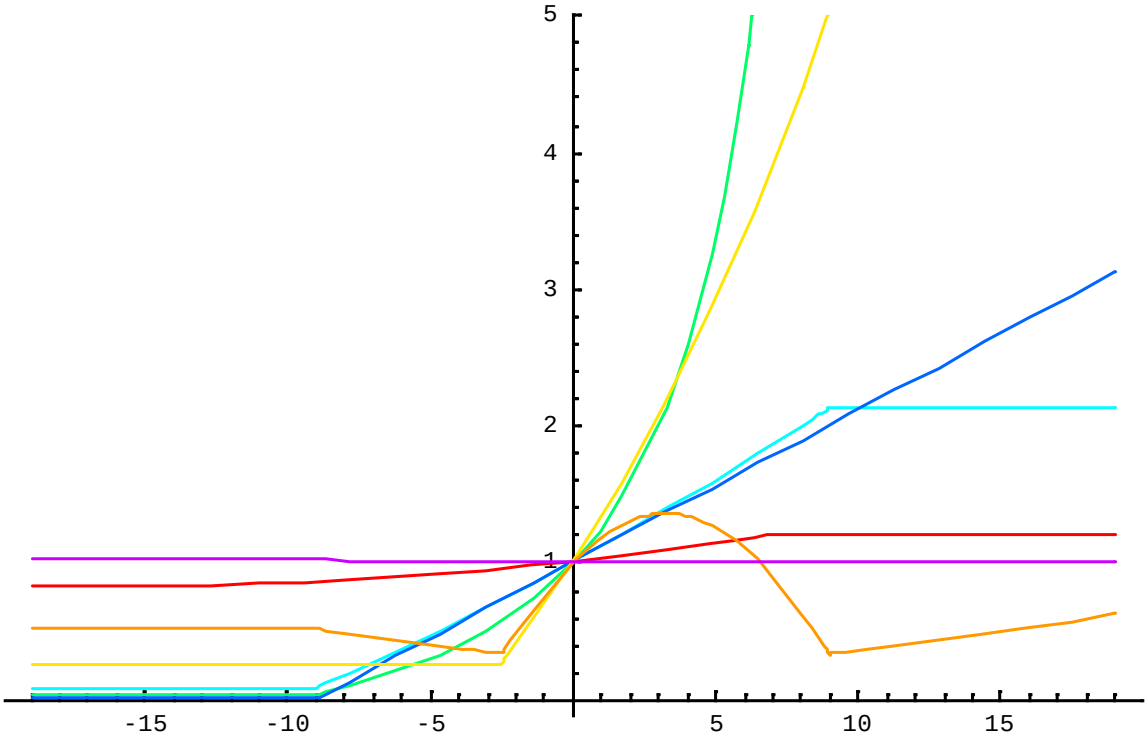

Figure. Comparative advantages to melee duel win ratio (w0).
Green: armor class, Cyan: attack bonus, Blue: hit points, Red: initiative, Yellow: damage bonus, Orange: power attack feat, Violet: expertise feat.